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Effect of initial conditions on two-dimensional free shear layers

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The influence of periodic perturbations on the development of two-dimensional free shear layers generated by a splitter plate was investigated in cases where the ratios of the two velocities u_1 and u_2 either side of the splitter plate were such that $0 < u_1/u_2 < 1$. Investigations were carried out in both a suction and a blower wind tunnel. Results show that even very weak periodic perturbations caused by the wind tunnel may cause significant nonlinear spreading in the downstream development of the shear layer, a behaviour which is also observed when the shear layer is deliberately excited. Other things being equal, the effect of the disturbance is greater when flow separation at the splitter plate is turbulent than when it is laminar.

No self-induced feedback frequencies were measured in the test section. All tonal components that were detected in the flow could be traced to external sources.

The influence of trailing-edge thickness on the shear-layer development is found to become significant when it exceeds 50 % of the sum of boundary-layer displacement thickness at the point of separation. As the trailing edge becomes thicker, the range over which the shear layer is self-similar is shifted farther downstream. This behaviour may be crucial for predicting the evolution of shear layers in high-speed flows having thin boundary layers at separation.

The momentum thickness criterion for estimating the development length of the flow as suggested by Bradshaw is shown to be insufficient for two-stream layers, where additional parameters, e.g. the trailing-edge geometry, have to be taken into account. Discrepancies between previously published observations of shear layers, as well as the considerable scatter in reported measurements, may therefore, to a large extent, be attributable to contamination of the experimental facility.

1. Introduction

The turbulent free shear layer – sketched in figure 1 – developing from laminar Blasius-type boundary layers on the splitter plate, is expected to approach a state of self-preservation well downstream of the region of laminar/turbulent transition at x_s. This condition can be achieved only if the local Reynolds number is sufficiently high and the flow has become independent of its initial conditions.

While for theoretical predictions these conditions are generally taken for granted, uncertainties arise concerning even the definition of Reynolds number in two-stream shear layers. Furthermore, in experiments the state of the flow plays an important role. Up to now there exists no reliable criterion for the development length necessary for the flow to become, if at all, independent of its initial conditions. Michalke (1965) predicted that the instability frequency of the laminar shear layer is governed by the

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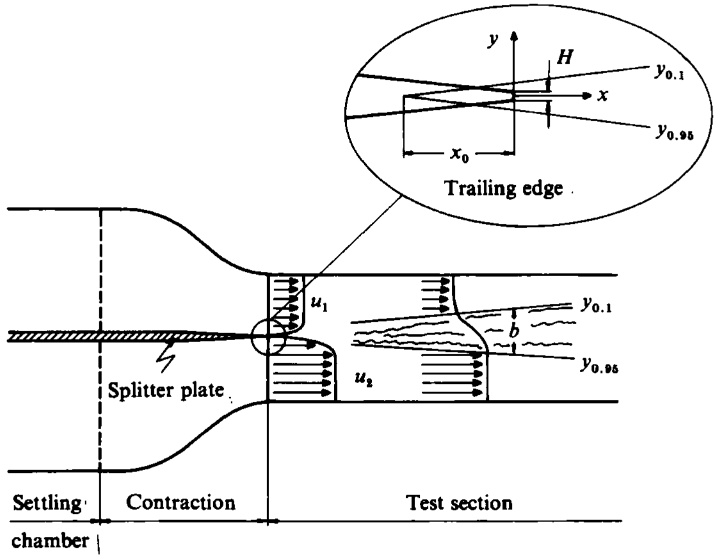

FIGURE 1. Flow configuration and notation.

exit boundary-layer momentum thickness and Bradshaw (1966) suggested that a fetch of $x_s/\delta_2 \approx 1000$ (δ_2 = momentum thickness of the separating boundary layer) is necessary for the shear layer to reach self-similarity. Even at downstream positions of $x_s/\delta_2 = 1300$ and 2800 respectively Browand & Latigo (1979) found different spreading rates for turbulent and laminar boundary-layer separation. So did Batt (1975) ($x_s/\delta_2 = 1500$), Oster, Wygnanski & Fiedler (1976) and Hussain & Zedan (1978*a*). These observations suggest either that the shear layers were not yet fully developed or would never become independent of the initial conditions.

Kleis & Hussain (1979) reported that for turbulent and laminar separation the flow achieved a unique self-preserving state for $x_s/\delta_2 > 2000$ while Birch (1976) found no influence of the momentum thickness at all. Obviously, Bradshaw's momentum thickness criterion applies only for ideal flow conditions, e.g. for a Blasius-type boundary layer at the trailing edge and single-stream mixing layers.

Crighton & Gaster (1976) found that small changes in the shape of the initial profile affect the stability characteristics significantly and Hussain & Zedan (1978*b*) included the boundary-layer turbulence level as an additional parameter of importance. Furthermore, Zaman & Hussain (1981) were able to suppress the shear-layer turbulence level by applying high-frequency forcing. Fiedler & Thies (1978) found that wind-tunnel frequencies may influence the shear-layer spreading behaviour, thus adding one more possible explanation for the reported discrepancies.

One of the basic problems in experiments is the exact definition and identification of the self-similar region. For this condition the apparent affinity of the mean velocity profiles may be misleading and on the basis of the theoretically expected linear spread one is often tempted to draw straight lines through too few experimental points, taking for scatter what in fact may be a nonlinear spread in a non-self-preserving regime. Constant maxima of u' intensity over a certain distance of the shear layer are also no reliable proof of a self-similar state of the flow. This is especially true for two-stream mixing layers as will be shown in §§3.1 and 3.3.

Periodically disturbed or forced shear layers show nonlinear spreading accompanied by a change in the turbulence structure, as for example investigated by Oster (1980)

Tunnel	Blower	Suction
Cross-section	0.5×0.6 m²	0.55×0.55 m²
Length of test section	1.5 m	2.0 m
Contraction ratio	1:7	1:5
Turbulence level in free stream	0.3 %†	< 0.4 %
Variation of $\bar{u}$ at test-section entrance	< 1 %	< 3.5 % in u_1
Angle of splitter plate	3°	4°
Height of splitter plate	≈ 0.3 mm	approx. 0.05 mm

† Depending on blower speed.

TABLE 1. Wind-tunnel data

and Mensing (1981). Depending on the forcing intensity, the randomness of the large-scale structures is reduced if the forcing wavelength is close to, or identical with, the naturally occurring characteristic wavelength. Then the coherence and lifetime of the large vortices is locally increased, resulting in a change of the spreading angle.

The present results prove that resonance and blower frequencies can be sufficient to influence the shear-layer development in the same way. The resulting nonlinear growth of the flow can be increased if the separating boundary layers are turbulent (trip wire). In this case, the periodic modulations forced upon the shear layer at the trailing edge are not randomized or damped by the strong stochastic processes of laminar turbulent transition. This effect becomes stronger for increasing velocity ratios r.

2. Apparatus, definitions

In the present investigations only shear layers with $u_1/u_2 = r > 0$ are considered. The experiments were carried out in a suction tunnel at Technische Universität Berlin and in a blower tunnel at Tel Aviv University, Israel. The characteristics of the wind tunnels are listed in table 1.

The blower tunnel (figure 2) was operated by two individual centrifugal blowers, one for each stream. The synchronous-motor-driven blower produced a frequency of about 58 Hz in the u_2 stream, while the d.c.-motor-driven blower generated no typical frequency. The wind tunnel had a resonance frequency of approximately 12 Hz. Zero pressure gradient $\mathrm{d}p/\mathrm{d}x = 0$ was achieved by flexible top and bottom walls of the test section. The suction tunnel (figure 3) was operated by a single axial blower. Velocity ratios were set by fitting sheets of foam rubber into the upper half of the settling-chamber inlets. In order to avoid separation at the leading edge of the splitter plate a perforated metal sheet was mounted as an upstream extension of the splitter plate, as seen in figure 3. Zero pressure gradient was adjusted with the top wall of the test section only. A weak blower frequency of about 35 Hz was observed in the suction tunnel.

For all measurements single-hot-wire probes were used. In the blower tunnel a hot-wire rake with up to ten hot wires was used for most of the measurements. Data acquisition and linearization were supported by digital computer. Momentum thicknesses, spreading rates etc. were calculated with Fortran programs. All computed results were graphically checked. The spread rates $\mathrm{d}b/\mathrm{d}x$ were calculated by the least-squares method in the considered flow range of the shear layer. This procedure was also applied when the spreading was obviously nonlinear in order to demonstrate

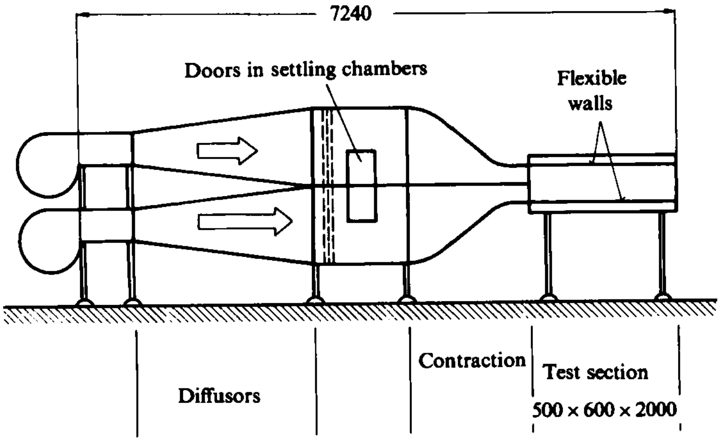

FIGURE 2. Suction tunnel.

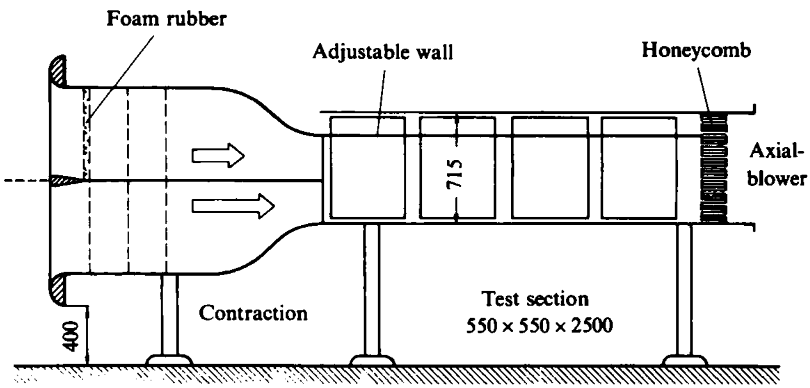

FIGURE 3. Blower tunnel.

and compare spreading rates which were calculated assuming linear spreading. In all measurements velocity u_2 was fixed at 13 m/s.

From the results of Liepmann & Laufer (1947) reference values for the spread rates $\mathrm{d}b/\mathrm{d}x = (y_{0.95}-y_{0.1})/(x-x_0)$ were calculated and used for comparison. For this purpose the Abramovich–Sabin relation,

$$\lambda = \frac{1-r}{1+r} = \frac{u_2-u_1}{u_2+u_1}, \tag{1}$$

was employed (see Birch & Eggers 1972) so that

$$\left.\frac{\mathrm{d}b}{\mathrm{d}x}\right|_{r>0} = \lambda \left.\frac{\mathrm{d}b}{\mathrm{d}x}\right|_{r=0}, \quad \frac{\sigma_0}{\sigma} = \lambda. \tag{2}$$

Following Brown & Roshko (1974) the maximum slope thickness,

$$\delta = (u_2-u_1)\left(\frac{\partial \bar{u}}{\partial y}\right)^{-1}_{\max}, \tag{3}$$

was used to calculate the spreading constant:

$$k = \frac{\delta}{x-x_0}\,\lambda, \tag{4}$$

which is then valid for all velocity ratios.

An integral measure for the local layer width is provided by the momentum thickness

$$\Theta = \int_{-\infty}^{\infty} \frac{\bar{u}-u_1}{u_2-u_1}\left(1-\frac{\bar{u}-u_1}{u_2-u_1}\right) \mathrm{d}y. \tag{5}$$

The spread parameter σ was evaluated in three different ways (see Dziomba 1981):
(*a*) via Görtler's (1942) first approximation

$$\sigma = \frac{\xi}{\eta}, \quad \text{with} \quad \eta = \frac{y-y_{0.5}}{x-x_0} \quad \text{and} \quad \frac{u(y)-u_1}{u_2-u_1} = \tfrac{1}{2}(1+\operatorname{erf}(\xi)); \tag{6}$$

(*b*) after Sabin (1963):

$$\sigma = \frac{x-x_0}{v_{0.922}-y_{0.5}}; \tag{7}$$

(*c*) after Korst & Chow (1962)

$$\sigma = \frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{\bar{u}-u_1}{u_2-u_1}\right)_{\max}. \tag{8}$$

Frequency spectra and correlations were measured with separate laboratory instruments.

3. Results

3.1. *Shear layer with disturbances*

A summary of the results of the shear-layer measurements with laminar separating boundary layers is given in table 2. The data of the corresponding boundary layers are presented in table 3.

The frequencies generated in the blower tunnel influenced the shear layer as will be shown below. The spreading of a typical shear layer with $u_1/u_2 \approx 0.41$ is depicted in figure 4(*a*). The maximum Reynolds number is $Re = \Delta u\, x/\nu = 8.5\times 10^5$. The spreading rate based on the velocity profiles for $800 < x < 1600$ mm was found to be $(y_{0.95}-y_{0.1})/(x-x_0) = 0.086$. In this x range the normalized u' intensities shown in figure 5 are constant, indicating self-similarity of the shear layer. The spreading rate is about 20% higher than the reference values.

The nonlinearity of the momentum-thickness distribution (figure 4(*a*), right) is less obvious in the isotachs (figure 4(*a*), left) but evaluation of the shear layer width $b = y_{0.1}-y_{0.95}$ qualitatively revealed the same tendency. The sudden change of the spreading angle at $x \approx 1300$ mm is a consequence of the forcing effect of the wind-tunnel resonance frequency of 12 Hz. In the frequency spectra taken at the u_1 edge of the shear layer (figure 6) this frequency is seen to be independent of the longitudinal position x marking it as an external frequency, while the natural large-scale frequencies become smaller by $f \sim 1/(x-x_0)$ as the shear layer grows. At a certain downstream stage the 12 Hz resonance frequency is close enough to the large-scale structures of the shear layer to 'lock in', leading to a stabilization of this wavelength in the flow as described above. At the beginning of this process the shear-layer spreading is increased, exceeding the natural value. It is followed by a region of reduced growth, indicating the existence of stabilized vortices (Fiedler & Mensing 1985). While this region of stabilized vortices is outside the test section the first part of the 'lock-in' occurs within the test section and causes the increase in shear layer width as seen in figure 4(*a*).

In an effort to prove this concept, the acoustic behaviour of the wind tunnel was

Tunnel	r $\frac{u_1}{u_2}$	λ $\frac{u-u_1}{u_2-u_1}$	Reference values σ	Reference values $\frac{db}{dx}$	σ Different methods of evaluation a	b	c	$\frac{db}{dx}$ $\frac{y_{0.1}-y_{0.9}}{x-x_0}$	$\frac{y_{0.1}-y_{0.95}}{x-x_0}$	x_0	k	$\frac{(\overline{u'^2})^{\frac{1}{2}}}{u_2-u_1}$	
Blower	0.41	0.42	26.3	0.071	22.5	24.0	21.4	0.078	0.086	+85	0.197	0.182	With disturbing frequencies.
Blower	0.41	0.42	26.3	0.071	27.6	30.1	26.1	0.064	0.070	−118	0.161	0.178	12 Hz eliminated
Suction	0.42	0.41	26.8	0.069	28.2	29.4	26.5	0.063	0.070	−148	0.163	0.176	—

TABLE 2. Spreading of the shear layer with laminar separation. $\sigma_0 = 11$; $(db/dx)_{r=0} = 0.169$ and reference values after Liepmann & Laufer (1947).

		$u_2 = 13$ m/s Trip wire		u_1		
		Without	With	5.2 m/s $r = 0.4$	7.9 m/s $r = 0.6$	3.9 m/s $r = 0.3$
Blower tunnel	δ_1	1.11	1.37	1.78	1.62	(2.05)
	δ_2	0.44	0.92	0.68	0.62	(0.79)
	H_{12}	2.54	1.49	2.62	2.61	—
Suction tunnel	δ_1	1.12	1.51	1.75	—	—
	δ_2	0.44	0.97	0.69	—	—
	H_{12}	2.55	1.55	2.53	—	—

TABLE 3. Splitter-plate boundary layer characteristics at trailing edge

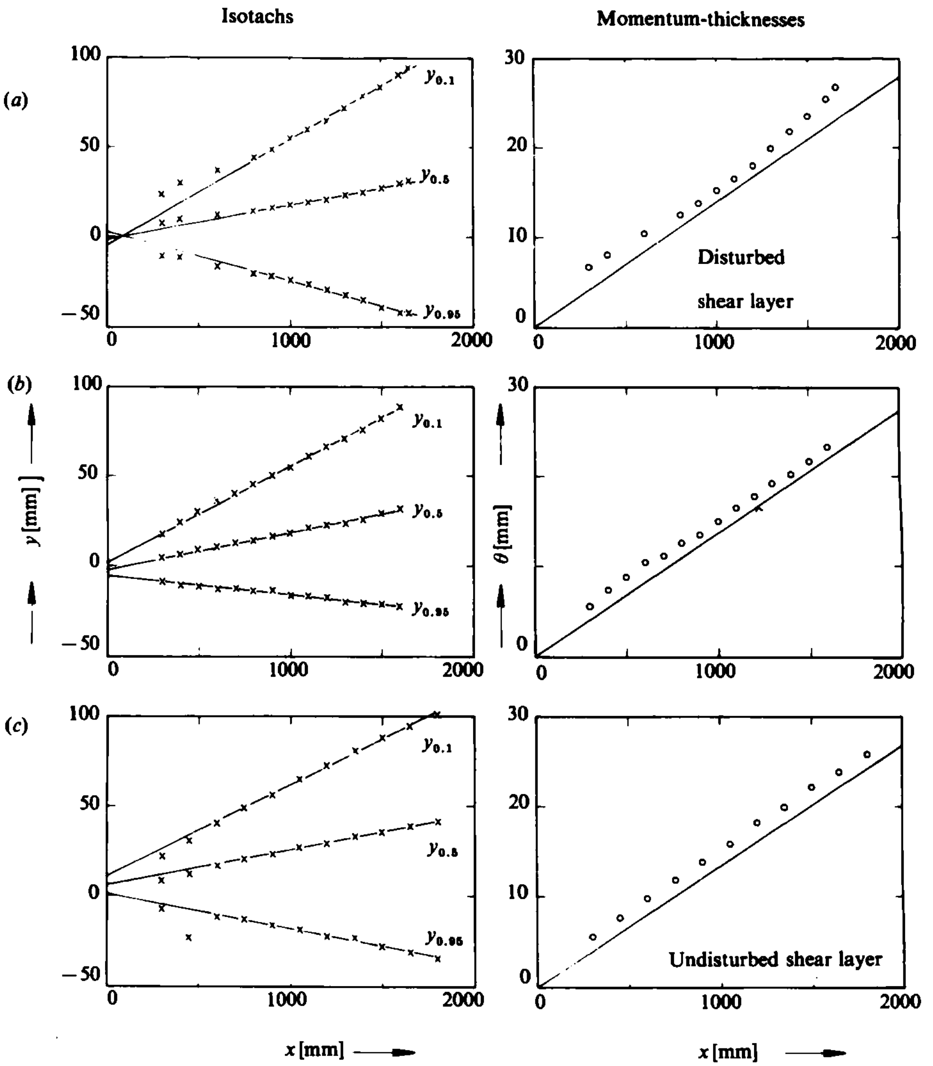

FIGURE 4. Spreading of shear layer for $u_1/u_2 \approx 0.41$: (*a*) blower tunnel with disturbances; (*b*) blower tunnel without disturbances; (*c*) suction tunnel. ——: reference slope as obtained in suction tunnel.

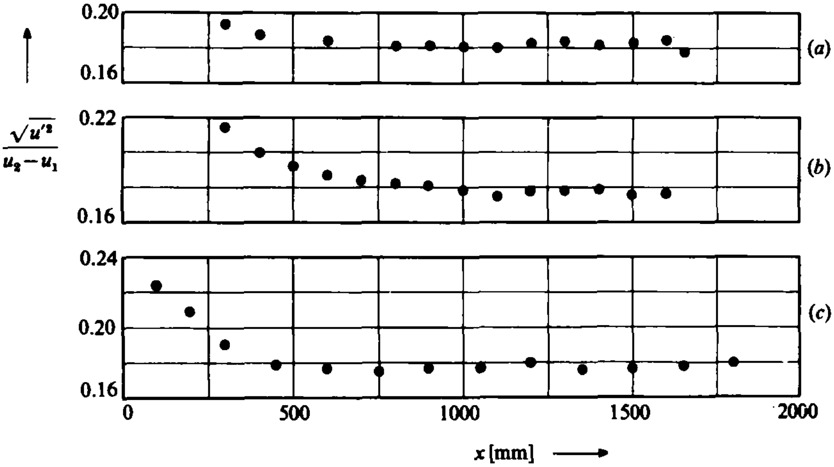

FIGURE 5. u' intensity maxima for $u_1/u_2 \approx 0.41$: (*a*) blower tunnel with disturbances; (*b*) blower tunnel without disturbances; (*c*) suction tunnel.

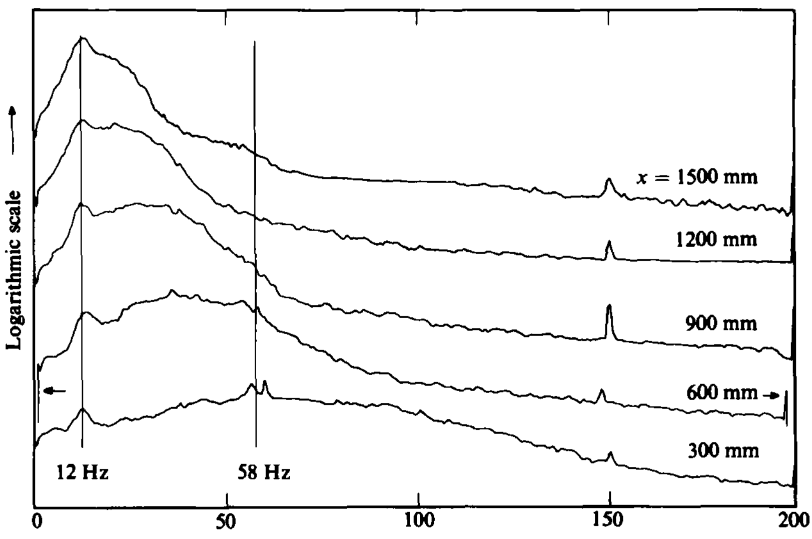

FIGURE 6. Frequency spectra in disturbed shear layer of blower tunnel (150 Hz frequency peak from mains.) $u_1/u_2 = r = 0.4$; $u_2 = 13$ m/s; no trip wire.

changed by replacing the solid wooden door of the settling chamber (see figure 2) by a 15 mm thick sheet of foam rubber. By this modification the mean-velocity profile at the nozzle outlet was not changed but the maximum velocity dropped by 10% and the turbulence level dropped to 0.2%.

As can be seen from the frequency spectra in figure 7 this modification of the tunnel eliminated the 12 Hz resonance frequency. The spreading rate calculated for the same longitudinal range as before dropped to $db/dx = 0.070$, being reasonably linear in the upper flow region (figure 4, centre row). This spreading rate is identical with the measurements in the suction tunnel, which was essentially free of disturbing frequencies, as is evident from figure 8. Note that all frequency spectra for the suction

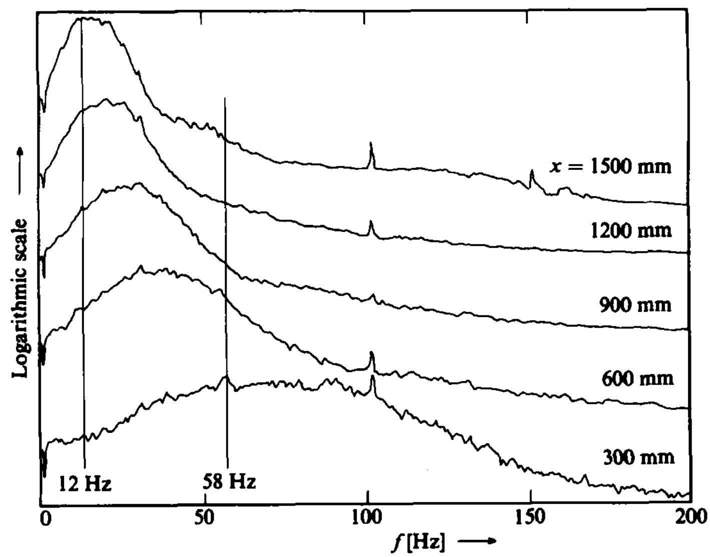

FIGURE 7. Frequency spectra after modification of blower tunnel, 12 Hz resonance frequency eliminated. $r = 0.4$; $u_2 = 11.5$ m/s; no trip wire.

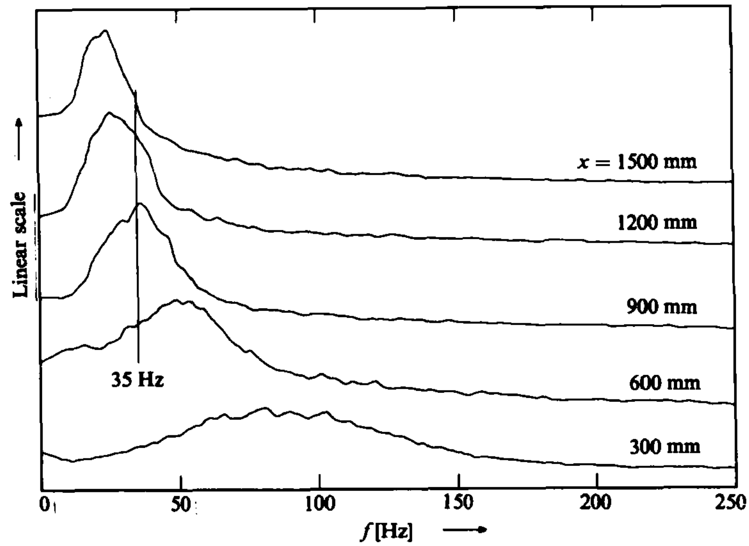

FIGURE 8. Frequency spectra of shear layer in suction tunnel. $r = 0.42$; $u_2 = 13$ m/s; no trip wire.

tunnel are plotted on a linear scale, while the spectra for the blowing tunnel are on a logarithmic scale. Since the latter shear layer showed good linearity and agreement with the reference value in table 2, its spreading gradient was added into the momentum-thickness graphs of all measurements for comparison.

These experiments confirm that wind-tunnel frequencies are able to influence the shear layer significantly. Recalling the fact that the spreading of a forced shear layer decreases strongly in the range of stabilized vortices, not only too high but also too low spreading rates may be found if the experimenter considers this region as the final self-similar part. Therefore, in disturbed shear layers, the calculated spreading rate

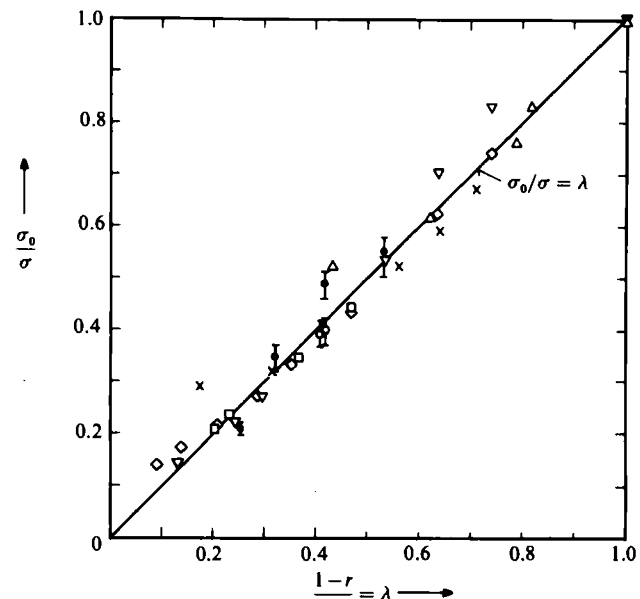

FIGURE 9. σ/σ_0 values for disturbed and undisturbed shear layer as obtained from present measurements compared with some data from literature. ◇ Miles & Shih (1968), $\sigma_0 = 9.3$; ▽ Spencer & Jones (1971), $\sigma_0 = 11.0$; □ Sabin (1963), $\sigma_0 = 11.0$; △ Yakovleskiy and × Zhestkov (Abramovich 1963), $\sigma_0 = 11.0$. Present measurements: ●, disturbed; ○, undisturbed. I, region of σ/σ_0 for different evaluation methods ($\sigma_0 = 11.0$).

depends not only on the velocity ratio but also on the convection velocity, the disturbing frequency and the test-section length. The typical scatter of spreading rates is demonstrated in figure 9 in which the calculated σ values are plotted versus velocity ratio λ. The spreading rates in the disturbed shear layer of the blower tunnel disclose a significant deviation from the ideal curve, while the measurement from the modified blower tunnel and from the suction tunnel are close to the ideal curve.

The importance of wind-tunnel disturbances is emphasized by the fact that the scatter of the present data is almost as large as the scatter of data found in the literature. It might be mentioned that an additional uncertainty in the comparison of spreading values is introduced if the σ value is calculated by different methods (see §2). The vertical bars in figure 9 represent the range of variations for the present measurements depending on the method of obtaining σ.

3.2. *Effect of a trip wire*

A trip wire near the trailing edge was used to produce a turbulent boundary layer on the u_2 side of the splitter plate. For all trip-wire measurements a wire of 1.5 mm diameter was mounted about 20 mm upstream of the trailing edge on the u_2 side of the splitter plate.

No wake effect of the trip wire was found in the boundary-layer profiles at the trailing edge. The boundary-layer data are given in table 3. Since for the same free-stream velocity the turbulent boundary layer is thicker than a laminar one, the shear layer downstream of the trailing edge will also be thicker in the early stage of development than the laminar one. This leads to a weaker gradient $d\bar{u}/dy$ and, consequently, to lower u' intensities and a smaller spreading angle. Further down-

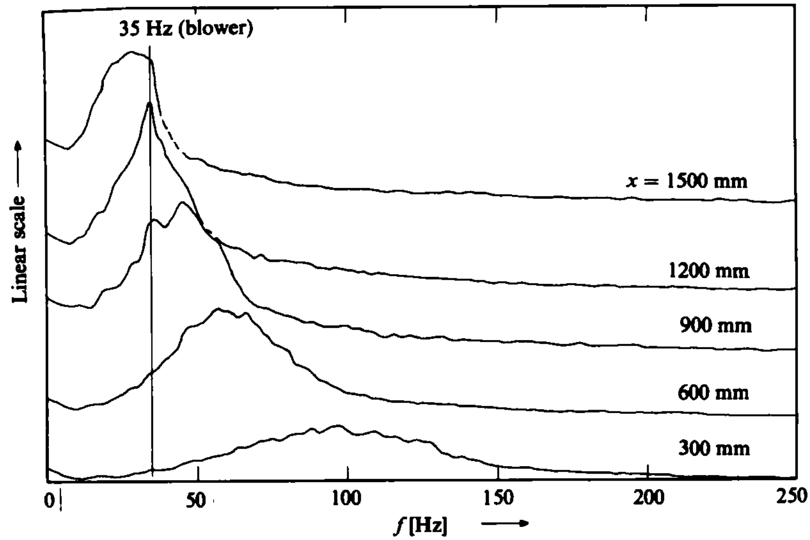

FIGURE 10. Frequency spectra of shear layer in suction tunnel with trip wire. 35 Hz blower frequency is amplified at $x = 900$ mm and $x = 1200$ mm (compare figure 8). $r = 0.42$; $u_2 = 13$ m/s.

stream, i.e. in the self-preserving region, the spreading rates for turbulent and laminar separation become, however, identical which is obvious from figure 15.

Furthermore it was found that in the presence of a trip wire periodic disturbances in the flow were amplified. This was first observed in the suction tunnel which was originally believed to be free of disturbing fequencies (figure 8). A sharp peak at 35 Hz was observed in the frequency spectrum at $x = 1200$ mm with a trip wire mounted (figure 10). This frequency is twice the blower r.p.m. The same effect was then observed in the blower tunnel where at $u_1/u_2 = 0.4$ a 58 Hz blower frequency showed in the spectrum (figure 11) when the trip wire was mounted. This frequency is clearly weaker in the frequency spectra of the laminar separating shear layer (figure 6). The autocorrelation functions (figures 12 and 13) in both tunnels again display a stronger correlation of longer duration for the 35 Hz blower frequencies in the case of tripped flow.

Further investigations revealed that it is not the trip wire as such, or its position on the splitter plate, that is responsible for this effect but the subsequent turbulent separation. This observation may be explained as follows. It is known that the free shear layer is most sensitive to periodic perturbations at or near its thinnest part, immediately downstream of the trailing edge. The rate of amplification, on the other hand, depends on the shape of the mean-velocity profile (Michalke 1965). While at turbulent separation the flow is comparatively smooth and statistically two-dimensional near the trailing edge, the transitional process following the separation of a laminar boundary layer is characterized by stronger randomization and strong stochastic modulation of the velocity profile leading to a reduced sensitivity or receptivity of the flow to a weak periodic perturbation. This is obviously the case for the 35 Hz blower frequency in the suction tunnel with laminar separation, which is more effective in the presence of a trip wire (figure 8). The disturbance is weakly amplified while travelling up to the point where it 'locks in' with the natural structure at $x \approx 1200$ mm (figure 10). From autocorrelation functions (figure 12) it is concluded that higher frequencies, e.g. the 58 Hz frequency in the blower tunnel, are more

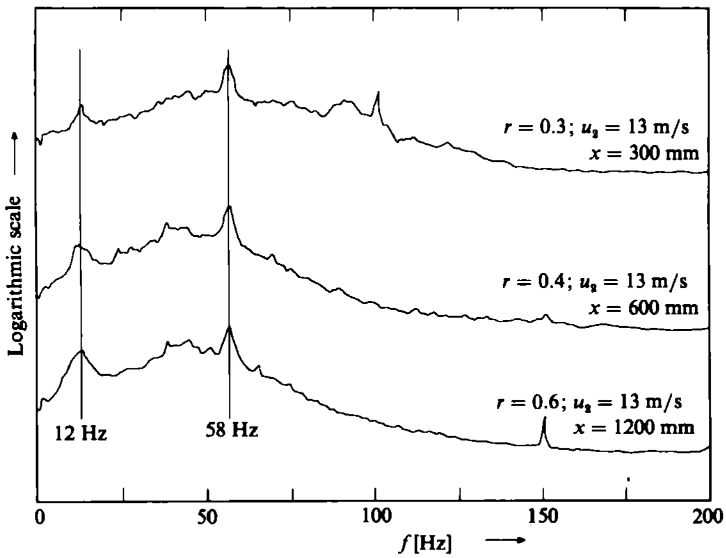

FIGURE 11. Frequency spectra in blower tunnel with trip wire with 12 Hz resonance frequency and 58 Hz blower frequency. The 58 Hz frequency is amplified compared to figure 6.

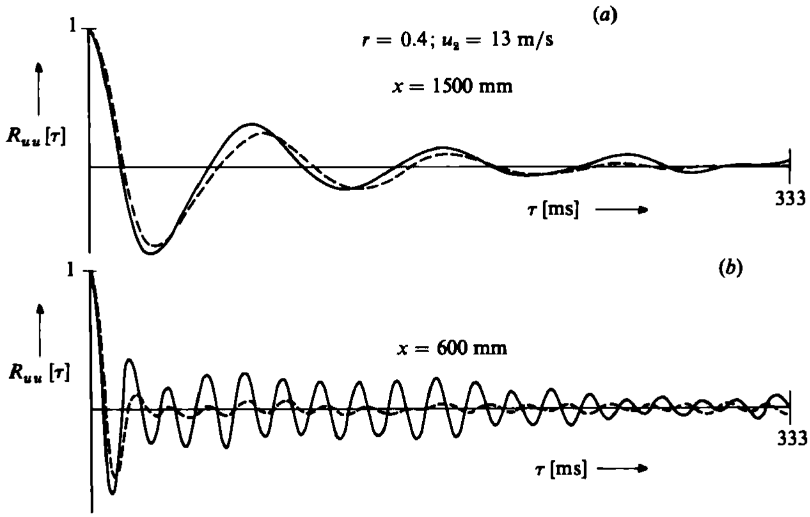

FIGURE 12. Autocorrelation in blower tunnel – influence of trip wire: (a) 12 Hz resonance frequency; (b) 58 Hz blower frequency. ---- without trip wire; —— with trip wire.

strongly suppressed by the transition process than lower frequencies, e.g. the 12 Hz frequency. The instability frequency of the laminar free shear layer was in the range of 600 Hz in the suction tunnel (see §3.3). It does not survive the transition process.

It might be mentioned that the 35 Hz blower frequency in the suction tunnel appeared in the spectra only at around $x = 1200$ mm, being considerably weaker than the disturbing frequencies in the blower tunnel (12 Hz and 58 Hz) which are present over the whole test-section length. Furthermore, the different scales of the frequency graphs, which are due to the different instruments used, exaggerate the 35 Hz perturbation.

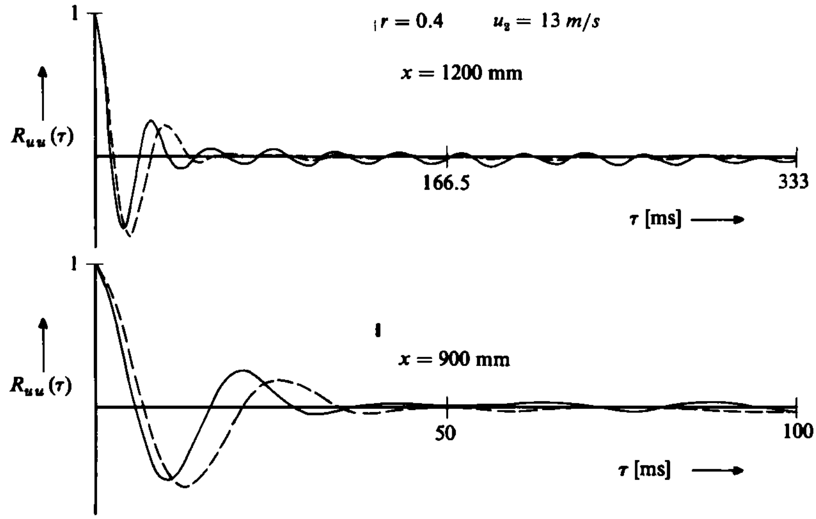

FIGURE 13. Autocorrelation in suction tunnel – influence of trip wire on 35 Hz blower frequency. ----, without trip wire; ——, with trip wire.

Because of the amplified influence of the disturbing frequencies at turbulent separation, the nonlinearity of the shear-layer spreading is increased. For the velocity ratio of 0.4 the spreading rate in the blower tunnel (figure 14) was measured to be $db/dx = 0.099$, thus exceeding the reference value in table 4 by 35%. Despite its strongly increased correlation coefficients, the 58 Hz blower frequency has very little effect on the spreading. For $r = 0.6$ only (figure 14) the nonlinearity at $x < 1000$ mm might be due to the 58 Hz frequency while the 'lock-in' for the 12 Hz frequency is already outside the test section. At $r = 0.6$ the spreading rate is slightly (7%) less than the reference value in table 4.

In the suction tunnel the spreading rates in the self-preserving regime for laminar and turbulent separation are almost identical, indicating the negligible effect of the 35 Hz frequency (see table 4 and figure 15).

The distribution of the u' intensities in the disturbed shear layer for $r = 0.305$ and 0.4 (figure 16) show an unusual tendency: after reaching a maximum they decrease again further downstream. A decrease of the u' intensities is a known feature of forced shear layers in the region of stabilized vortices (Oster 1980). With respect to the present measurements this behaviour might be understood as a strong forcing effect of the 12 Hz resonance frequency.

A more detailed account of the results described in §§3.1 and 3.2 is given by Dziomba (1981).

3.3. *Effect of trailing-edge thickness*

The momentum thickness of the separating boundary layers is considered to be an essential parameter for the development length of the free shear layer. For two-stream mixing layers the trailing-edge thickness represents an additional parameter.

The experiments described below were carried out in the two-stream mixing layer of the suction tunnel. For the measurements the arrangement of screens was changed and honeycombs were placed in the settling-chamber inlet to reduce the free-stream turbulence level to less than 0.2%.

At the onset of the laminar/turbulent transition, periodic oscillations occur in the

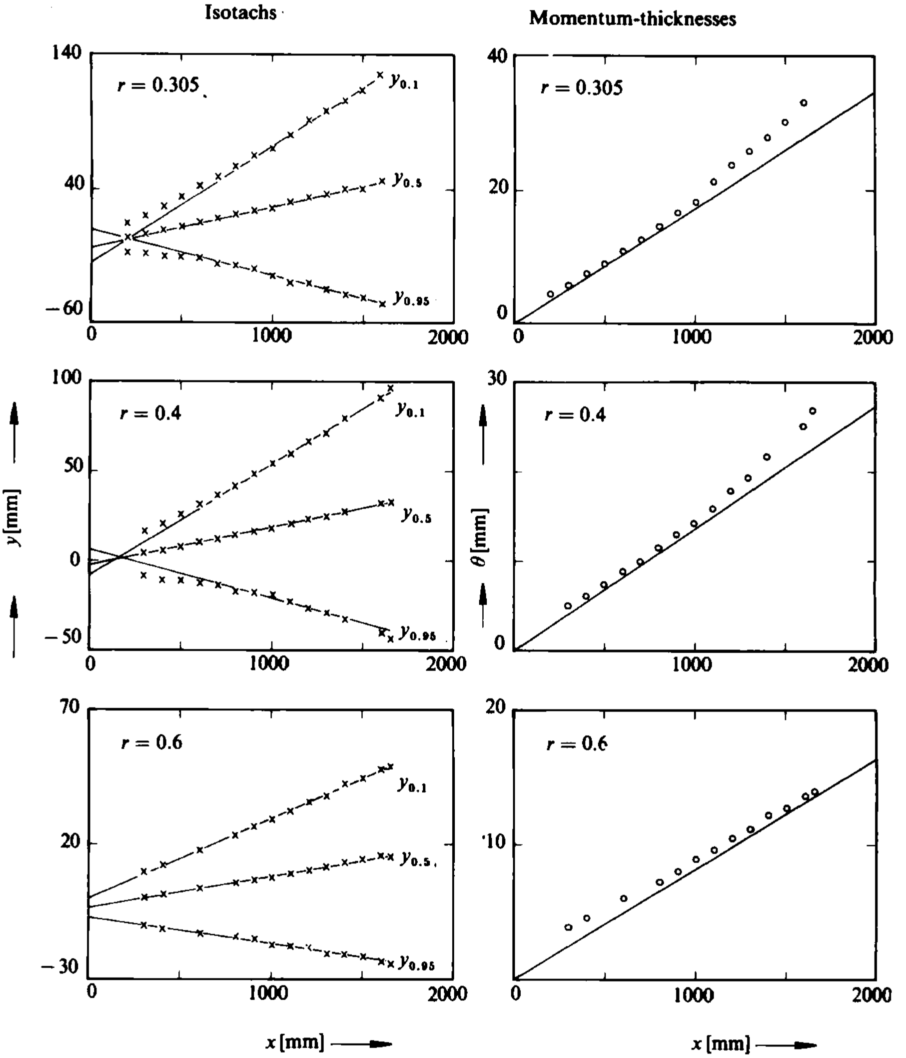

FIGURE 14. Spreading of shear layer in blower tunnel with disturbances and trip wire.

free shear layer (figure 17). According to Freymuth (1966) this instability frequency is governed by the momentum thickness of the trailing-edge boundary layers. This was verified in the present measurements for two-stream mixing layers by extending the splitter plate by 600 mm and thus increasing the boundary-layer momentum thickness by 25 %. With this extension the instability frequency of the shear layer was reduced by 23 % while the Strouhal number remained constant as predicted.

The boundary-layer thickness was then varied by changing the free-stream velocity. In this case

$$f_i = ku^{\frac{3}{2}} \tag{9}$$

(f_i = instability frequency).

From measurements with $u_1/u_2 \approx 1$ the constant in this relation was found to be $k = 24\ (\mathrm{sec\ m^{-3}})^{\frac{1}{2}}$. A comparison between (9) and the measurements is given in

Tunnel	r $\frac{u_1}{u_2}$	λ $\frac{u-u_1}{u_2-u_1}$	Reference values σ	Reference values $\frac{\mathrm{d}b}{\mathrm{d}x}$	σ a	σ b	σ c	$\frac{\mathrm{d}b}{\mathrm{d}x}$ $\frac{y_{0.1}-y_{0.9}}{x-x_0}$	$\frac{\mathrm{d}b}{\mathrm{d}x}$ $\frac{y_{0.1}-y_{0.95}}{x-x_0}$	x_0	k	$\frac{(\overline{u'^2})^{\frac{1}{2}}}{u_2-u_1}$	
Blower	0.305	0.53	20.7	0.089	16.1	17.3	15.4	0.109	0.121	+200	0.217		With disturbing frequencies
Blower	0.40	0.43	25.7	0.073	19.2	21.0	18.7	0.088	0.099	+260	0.223		
Suction	0.41	0.42	26.8	0.071	27.8	28.9	25.9	0.063	0.071	+100	0.164	0.180	—
Suction	0.60	0.25	44.0	0.042		52.8	47.2	0.035	0.039	−192	0.151		With disturbing frequencies

TABLE 4. Spreading of the shear layer with turbulent separation (trip wire). $\sigma_0 = 11$; $\mathrm{d}b/\mathrm{d}x\,|_{r=0} = 0.169$ and reference values after Liepmann & Laufer (1947).

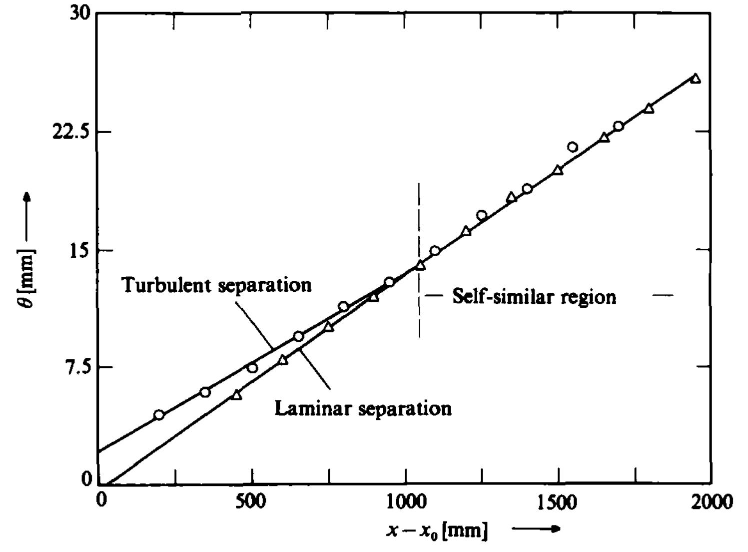

FIGURE 15. Longitudinal development of momentum thickness for laminar and turbulent separation in suction tunnel: $r = 0.41$ (virtually undisturbed flow).

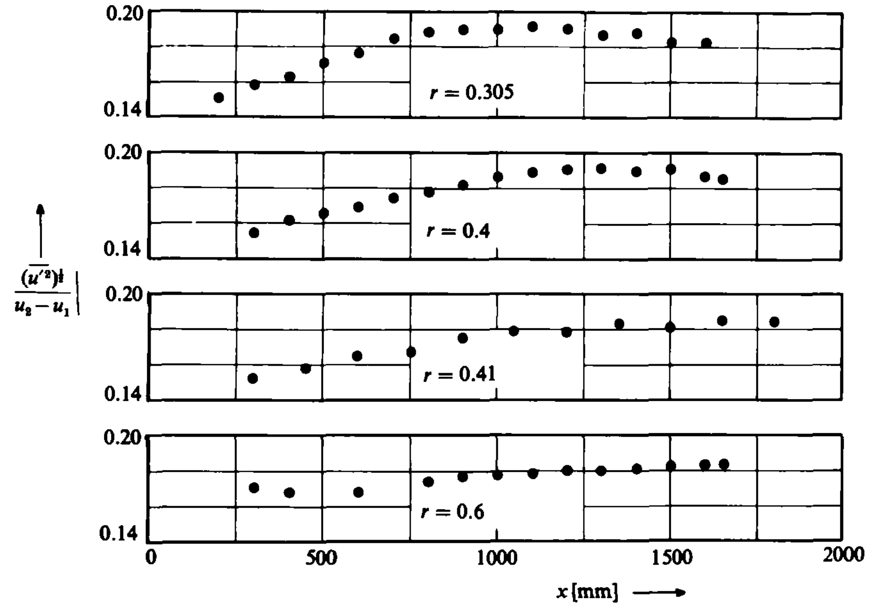

FIGURE 16. u' intensity maxima versus x for disturbed shear layer in blower tunnel and for shear layer in suction tunnel (3rd from top). Turbulent separation.

figure 18 for $r = 0.95$, 0.70 and 0.44, where for velocity ratios less than unity the following equation was applied:

$$f_1 = \tfrac{1}{2}[f_{r=1}(u_1) + f_{r=1}(u_2)]. \tag{10}$$

The good agreement between theoretical and experimental data proved the influence of the boundary layers on the instability frequency of free shear layers with $r > 0$.

Equation (10) holds only if the thickness of the trailing edge is small compared to

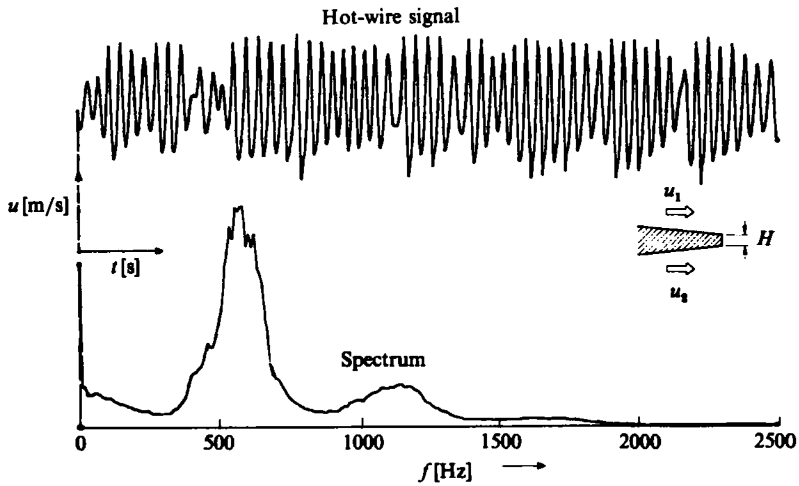

FIGURE 17. Instability frequency of laminar shear layer for thin (original) leading edge: $x = 40$ mm; $y = 0$ mm; original trailing edge $H \approx 0.05$ mm.

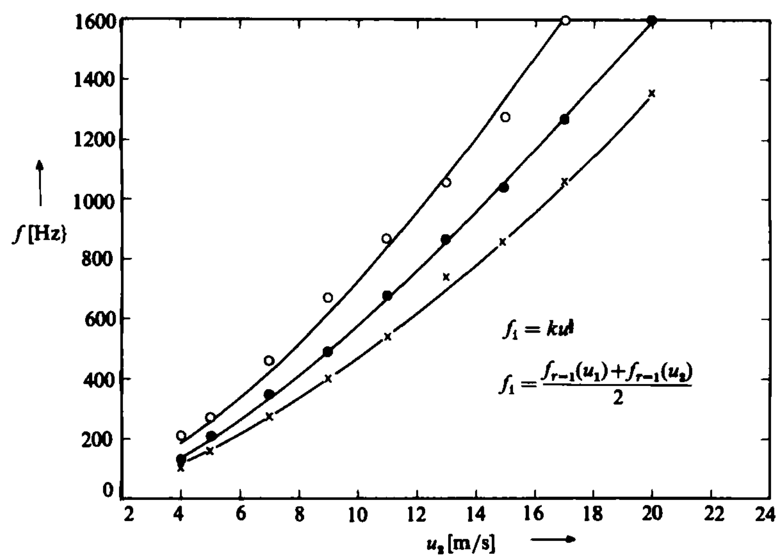

FIGURE 18. Dependence of instability frequency of the laminar shear layer on free-stream velocities. Measurements: ×, $r = 0.44$; ●, $r = 0.7$; ○, $r = 0.95$ ($k = 9.4$).

the displacement thickness of the separating boundary layers. In a next step, the trailing-edge thickness H was varied using an appropriate strip of thin sheet metal, which was attached to the upper part of the splitter plate, its upstream edge glued to the plate surface and faired with thin scotch tape. With increasing edge thickness the influence of the separating boundary layers is reduced and the instability frequency of the wake becomes dominant. In figure 19 measured wake frequencies are compared with the theoretical solution by Michalke & Schade (1963) who used a trapezoidal wake profile. The experimental results presented in figure 19 were obtained at constant velocity resulting in a constant boundary-layer profile. With the maximum Strouhal number of 0.14 and the highest frequency measured 720 Hz the constant b was found to be 1.75 mm for the present measurements. The good

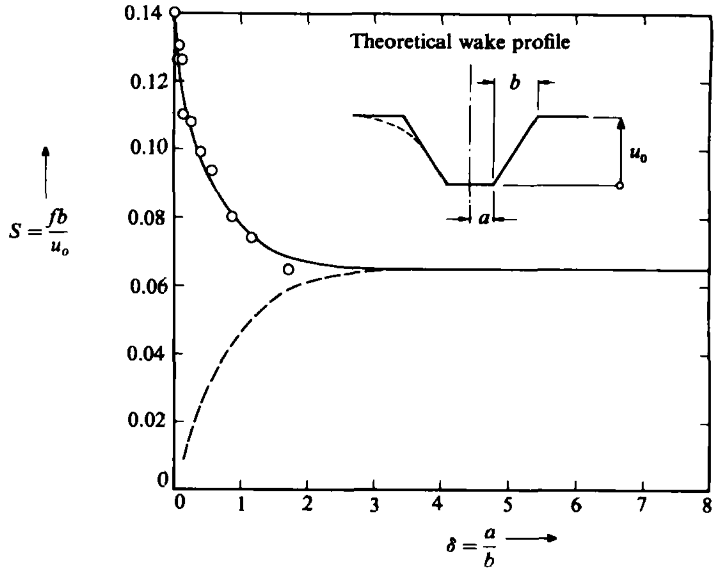

FIGURE 19. Wake frequencies as measured near separation for various heights of trailing edge $H \triangleq 2a$. Comparison with theoretical prediction of Michalke & Schade (1963) for trapezoidal wake profile: ○, measurements; ——, asym. perturbation (theory); ----, sym. perturbation (theory).

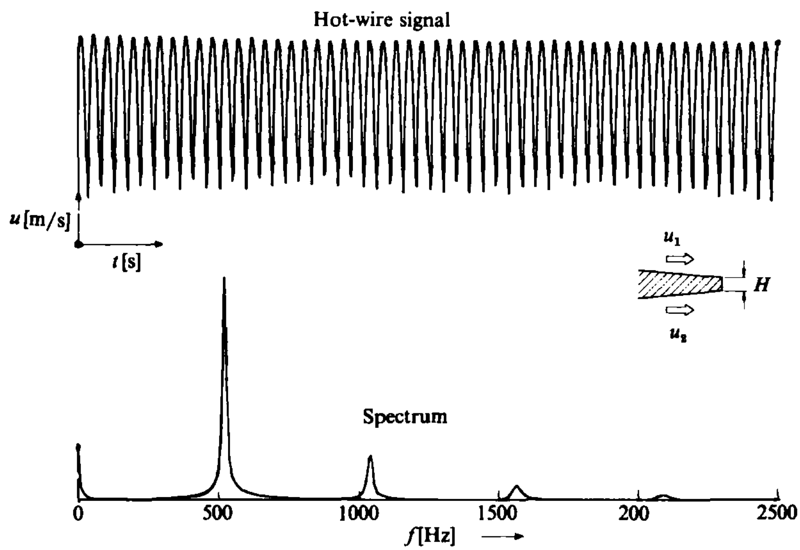

FIGURE 20. Instability of laminar shear layer for thick trailing edge: $x = 6.0$ mm; $y \approx u$, edge; $H = 2.0$ mm.

agreement between theory and experiment underlines the importance of the trailing-edge thickness for two-stream mixing layers.

For increased edge thickness the wake frequency became more stable, as can be seen from comparing figures 17 and 20. At the same time, the point of maximum amplitude moved towards the trailing edge, feeding its oscillations back into the splitter-plate boundary layers.

Results for a limited number of spreading-rate measurements are listed in table 5.

H (mm)	f (Hz)	δ_1 (mm)	δ_2 (mm)	$\Sigma\,\delta_1$ (mm)	$\dfrac{H}{\Sigma\,\delta_1}$	$\dfrac{db}{dx}$	x_0 (mm)	k	$\dfrac{(\overline{u'^2_{max}})^{\frac{1}{2}}}{u_2-u_1}$	
0.05	600	1.6 (u_1) 1.2 (u_2)	0.74 0.47	2.8	1.8 %	0.063	−250	0.155	0.172	BL's laminar
0.075	660	1.5 1.14	0.68 0.44	2.6	2.9 %	—	—	—	—	BL's laminar
0.5	670	1.45 1.10	0.66 0.41	2.6	20 %	—	—	—	—	BL's laminar
1.0	684	1.40 0.85	0.63 0.41	2.3	44 %	0.058	−43	0.142	0.172	BL's laminar
2.0	640	—	—	—	90 %	0.050	−100	0.125	0.170	BL's laminar
2.0	490	—	—	—	—	0.055	−150	0.135	0.170	u_2-BL nominally laminar; u_1 laminar
2.0	No discrete frequencies	1.32 1.37	0.61 0.90	2.69	77 %	0.058	+10	0.145	0.171	u_2-BL turbulent; u_1-BL laminar

TABLE 5. Shear-layer data for different initial conditions and $r = 0.44$ ($\lambda = 0.38$), $u_2 = 12$ m/s. Expected spreading rate according to Liepmann & Laufer (1947):

$$\frac{y_{0.95}-y_{0.1}}{x-x_0} = 0.065$$

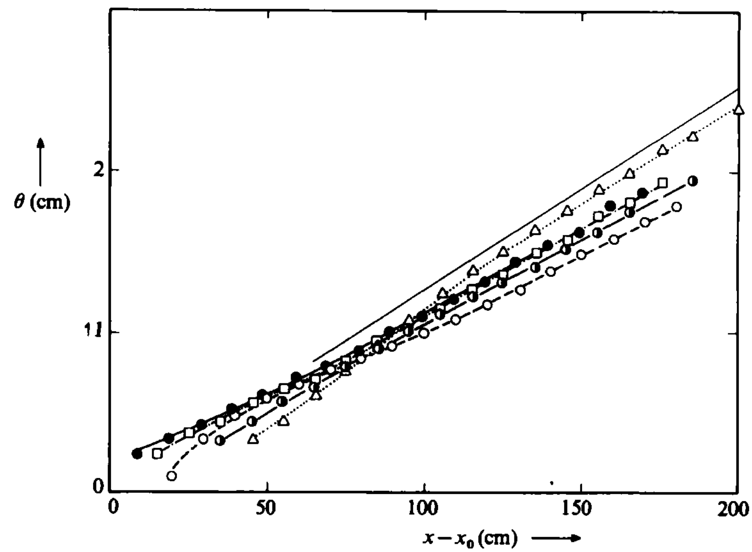

FIGURE 21. Momentum thickness distribution versus $(x-x_0)$ for various conditions at separation. ——, Slope after Liepmann & Laufer (1947).

	H	Boundary layer	x_0
△······△	< 0.05 mm	Laminar	−25 cm (original edge)
□—·—□	1 mm	Laminar	−4.3 cm
○---○	2 mm	Laminar	−10 cm
◑——◑	2 mm	Nom. lam.	−15 cm
●——●	2 mm	Turbulent	+1 cm

As the trailing-edge thickness H is increased the spreading of the shear layer decreases, as figure 21 shows. The same tendency is evident in the u' intensity maxima distribution. Along the downstream distance x in figure 22 the curves are falling further below the final value of 0.172 as the edge thickness is increased. For $H < 0.1$ mm the minimum is close to 0.170 while it drops to 0.152 for $H = 1$ mm and to 0.142 for $H = 2$ mm. This behaviour might be a consequence of the increased displacement of the separated boundary layers which results in a lower velocity gradient $\mathrm{d}u/\mathrm{d}y$ in the early stage of the shear-layer development. It is on the other hand quite reminiscent of Zaman & Hussain's (1981) observation of turbulence suppression by controlled excitation, which was reported to be most effective at somewhat higher Strouhal numbers than that for the natural shear-layer instability. Indeed the wake effect of the splitter plate provides a strong and regular forcing at slightly higher Strouhal number and visualization shows that for increasing values of H, i.e. for stronger wake effects, the instability vortices are enhanced and shifted upstream. Further downstream the u' intensity curves finally recover, indicating that the influence of the initial conditions has vanished and that the shear layer is returning back to a natural behaviour. It is thus clear that as the trailing edge becomes thicker the self-similar range of the shear layer is shifted further downstream.

The shedding frequencies given in table 5 rise with increased edge thickness contrary to theoretical expectation but in agreement with the decreasing momentum thickness of the boundary layers. The decrease is due to technical changes necessary for increasing H. Only for $H > 1.0$ mm do the frequencies finally drop. This indicates

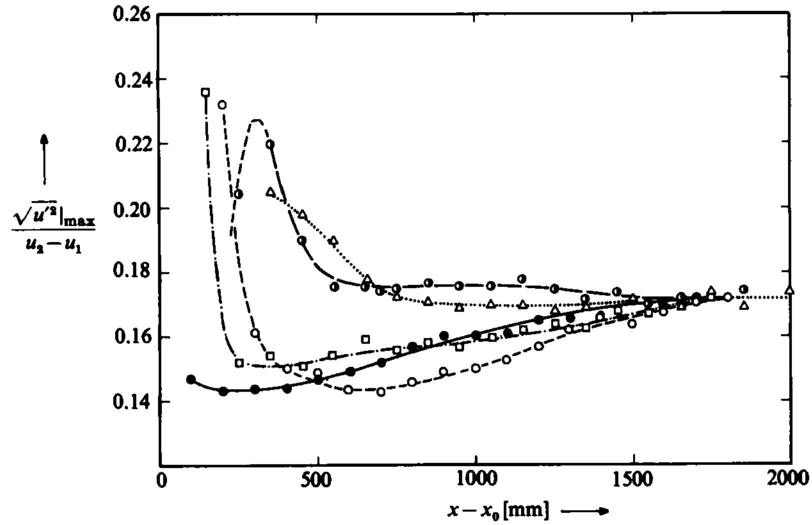

FIGURE 22. Maxima of u' intensities versus $(x-x_0)$. Symbols as in figure 21.

a critical ratio of edge thickness H to displacement thickness δ_1 of $(H/\delta_1)_{crit} = 0.5$. For this ratio the error in the spreading rate compared with the reference value of 0.065 (around 10 %). Remaining below the 50 % mark is therefore recommended to avoid serious trailing-edge effects or significantly longer development lengths, respectively.

In case of a turbulent u_2 boundary layer (trip wire) the effect of a 2 mm thick trailing edge was found to be reduced. While for laminar separation the spreading rate dropped by about 20 % compared with the $H > 0.1$ mm case, it was 10 % off when a trip wire was mounted. The lower $(H/\delta_1)_{crit}$ ratio of 0.77 and the higher initial turbulence level appear to be responsible for this behaviour. With laminar boundary layers the initial turbulence level also seems to have some importance to the development of the free-shear layer. This was observed while an incorrectly mounted 2 mm trailing edge caused turbulence spots in the basically laminar boundary layer. Following a suggestion of Hussain & Zedan (1978*a*) it might therefore be considered to be only nominally laminar. In this case the u' intensities soon reach constant maximum values of 0.176, implying early self-similarity (figure 22). At the same time the spreading rate is still more than 10 % below the $H \rightarrow 0$ case which, however, is less severe than in the completely laminar $H = 2$ mm case.

Similarly, in a measurement with an artificially increased turbulence level in the free stream of 2 % and, therefore, only nominally laminar boundary layers the maxima of the u' intensities very soon reached a constant level while the spreading was about 10 % above that of the low-turbulence case. Similar results were reported by Hussain & Zedan (1978*b*).

These examples demonstrate that constant u' intensities are also not reliable indicators for the self-similarity of the shear layer. It is furthermore concluded that, in addition to the momentum thickness of the separating boundary layers, the trailing-edge thickness and the turbulence level of the boundary layers are important parameters for the development of the shear layer in real flows.

4. Conclusions

The initial conditions, i.e. the conditions the flow meets upstream or immediately at the nozzle outlet, play an important role in the development of the free shear layer. One of the most crucial influences is a periodic oscillation forced upon the shear layer immediately downstream of the trailing edge, i.e. at its most sensitive region. The forcing might be applied by a loudspeaker (Mensing 1981; Bechert 1982; Wygnanski, Oster & Fiedler 1979) or by a vibrating flap (Oster 1980) if done deliberately. But a vibrating splitter plate (Pui & Gartshore 1979), blower frequencies (Drubka & Nagib 1981) or wind-tunnel resonance frequencies (Fiedler & Thies 1978) are able to affect the flow in a similar, but more moderate, way. The shear layer reacts on this periodic disturbance only if the excited waves are close or identical to the natural wavelength in the flow. In this case the corresponding structures (vortices) become stabilized over a certain distance while the spreading angle of the shear layer deviates from the linear. The coherence of the flow structure is increased.

This raises the issue of the structure of the natural undisturbed shear layer. From the present results it is concluded that the discrete coherent vortex, represented by a sharp peak in the frequency spectrum, is not the typical constituent of the free mixing layer. More likely, it is a limited-flow region in which the large-scale motion is somehow correlated. Only in the case of artificially added determinism (forcing) do these structures become organized and form discrete vortices.

This model contradicts to some extent some earlier publications, e.g. the paper of Brown & Roshko (1974). In their shear layer they found a spreading factor of $k = 0.181$ as compared with 0.161 by Liepman & Laufer (1947) and 0.163 from the present measurement in the undisturbed shear layer. Since their value is closer to the values 0.175 or 0.197 found for the undisturbed mixing layers in the present measurements it might be suspected that their shear layer was either disturbed or not fully developed.

Winand & Browand (1974) reported low u' intensities (0.12) in their water shear layer while the frequency spectra displayed a second maximum, indicating an external forcing frequency. While they approximate the momentum-thickness distribution by a straight line their data clearly show a nonlinear distribution.

The autocorrelation functions measured by Dimotakis & Brown (1976) revealed 'time lags that are longer than any obviously relevant timescale'. This effect is typical for forced shear layers. These authors, however, connect this phenomenon with a possible feedback in the test section. No such feedback effect was found in the present measurements. Instead, it turned out that all stable frequencies could be related to external sources such as resonance or blower frequencies.

As a result of unintended forcing the spreading rate was found to be more than 20% off the undisturbed rate. This deviation is further increased when a trip wire is mounted on the splitter plate. Because the laminar–turbulent transition does not take place in the free shear layer but on the splitter plate when a trip wire is used, the modulation forced upon the flow at the trailing edge is not weakened or cancelled by the transition process. Thus the effect of forcing is increased.

In the undisturbed shear layer the spreading rates for laminar and turbulent separating boundary layers are found to be identical in the self-preserving regime, but with necessarily different virtual origins.

While disturbing frequencies can influence the shear layer at almost any downstream position, only the developing region is influenced by the splitter-plate boundary layers or the trailing-edge thickness. With growing edge thickness the wake character of the near-field flow is strengthened and the instability frequency of the laminar shear layer is replaced by the shedding frequency of the wake. Assuming that a unique self-similarity exists, its onset is shifted downwards significantly with increasing trailing-edge thickness. This effect will play a minor role in low-speed wind tunnels but as the boundary layers get smaller, e.g. in high-speed or water tunnels, and as the test sections become shorter it will become a crucial parameter, since there is a physical limitation for milling a trailing edge.

The developing region of the shear layer is also affected by the turbulence level of the separating boundary layers. A laminar boundary layer containing turbulent spots leads to an early constancy of the u' intensities while the spreading rates may still be below or above the rate measured for truly laminar boundary layers. Therefore constant u' intensity maxima are not a reliable indicator for a self-similar shear layer. An observed Reynolds number dependence of a mixing layer might, in fact, be attributed to the Reynolds number dependence of the splitter plate boundary layers.

In conclusion, it can be stated that only ideal laminar or turbulent splitter-plate boundary layers lead to a development of the shear layer as expected from theory. Each external disturbance in the initial range changes the shear-layer characteristics in the developing region as compared to the ideal case. Thus, the realization of an experimental shear layer appears much more difficult than might have been expected heretofore.

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